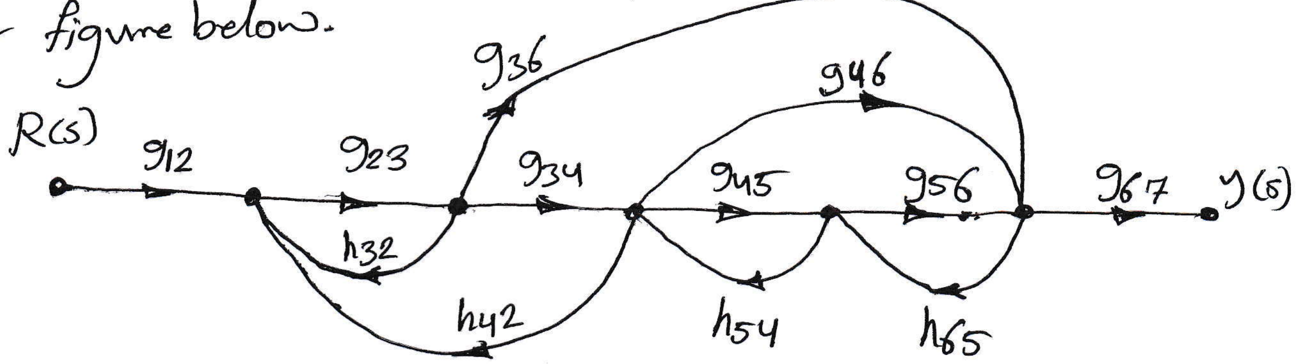
EX. Find the transfer function $\frac{Y(s)}{R(s)}$ by Mason's gain formula for figure below.



1 - Forward paths

$$M_1 = g_{12}\, g_{23}\, g_{34}\, g_{45}\, g_{56}\, g_{67}$$

$$M_2 = g_{12}\, g_{23}\, g_{36}\, g_{67}$$

$$M_3 = g_{12}\, g_{23}\, g_{34}\, g_{46}\, g_{67}$$

2- Loops

$$L1 = g_{23}\, h_{32}$$

$$L2 = g_{23}\, g_{34}\, h_{42}$$

$$L3 = g_{45}\, h_{54}$$

$$L4 = g_{56}\, h_{65}$$

$$L5 = g_{46}\, h_{65}\, h_{54}$$

$$L6 = g_{23}\, h_{36}\, h_{65}\, h_{54}\, h_{42}$$

3- Two non touching loops.

$$P_{12} = L_1 L_3 = g_{23}\, g_{45}\, h_{32}\, h_{54}.$$

$$P_{22} = L1\, L4 = g_{23}\, g_{56}\, h_{32}\, h_{65}.$$

$$P_{32} = L_1 L_5 = g_{23}\, g_{46}\, h_{32}\, h_{65}\, h_{54}.$$

4- Three non touching loops.

$$P_{m3} = P_{m4} = 0.$$

Now the determinant of the flow graph.

$$\Delta = 1-(L_1+L_2+L_3+L_4+L_5+L_6)+P_{12}+P_{22}+P_{32}$$

$$= 1-(g_{23}h_{32}+g_{23}g_{34}h_{42}+g_{45}h_{54}+g_{56}h_{65}+ g_{46}h_{65}h_{54}+g_{23}h_{36}h_{65}h_{54}h_{42})+ (g_{23}g_{45}h_{32}h_{54}+g_{23}g_{56}h_{32}h_{65}+ g_{23}g_{46}h_{32}h_{65}h_{54}).$$

Δ_1 : value of Δ which is non touching with M_1. all loop L_1 to L_6 have at least one node in common with M_1,

$$\Delta_1 = 1$$

Δ_2 : value of Δ which is non touching with M_2. the loop L_3 is not having any common node with M_2.

$$\Delta_2 = 1 - L_3 = 1 - g_{45}h_{54}$$

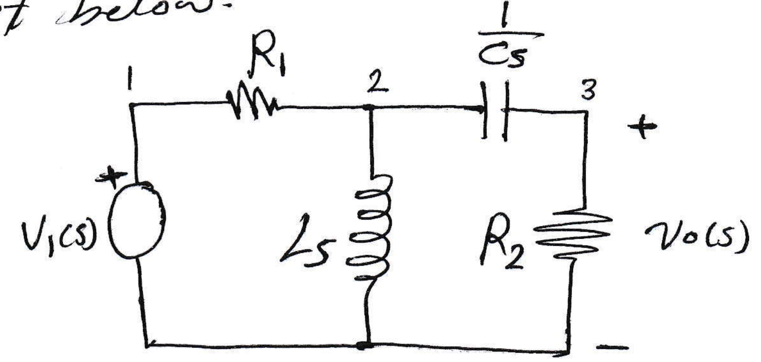
Δ_3 : value of Δ which is non toching with M_3.

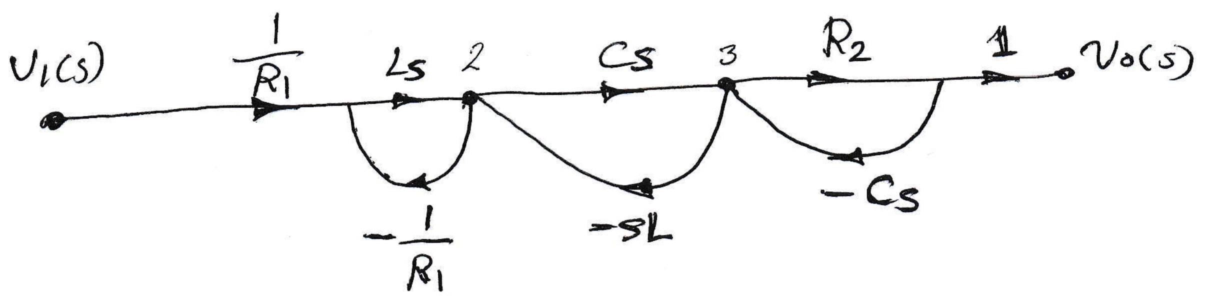
$$\Delta_3 = 1$$

Applying Mason's gain formula, we have

$$T(s) = \frac{Y(s)}{R(s)} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta}$$

EX Find the transfer function $\frac{V_o(s)}{V_i(s)}$ by Mason's gain formula for the circuit below.





1- Forward path

$$M_1 = \frac{1}{R_1} \cdot SL \cdot Cs \cdot R_2$$

$$= \frac{LCS^2 R_2}{R_1}$$

2- loop gain

$$L1 = -\frac{LS}{R_1}.$$

$$L2 = CS(-SL) = -S^2LC.$$

$$L3 = -R_2 CS.$$

3- two non touching loops.

$$P_{12} = L_1 \cdot L_3$$
$$= -\frac{SL}{R_1}(-R_2 C S)$$
$$= \frac{R_2}{R_1} \cdot LCS^2$$

4- the determinant of the flow graph.

$$\Delta = 1 - (L_1 + L_2 + L_3) + P_{12}$$
$$= 1 + \left(\frac{LS}{R_1} + S^2LC + R_2CS\right) + \frac{R_2}{R_1}LCS^2$$

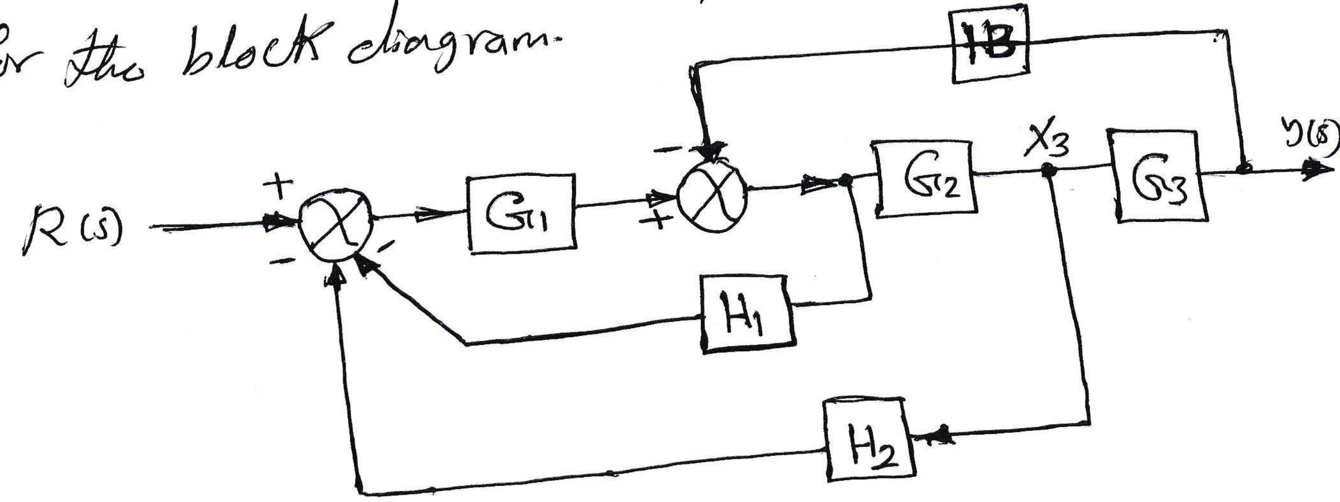
$\Delta_1 = 1$.

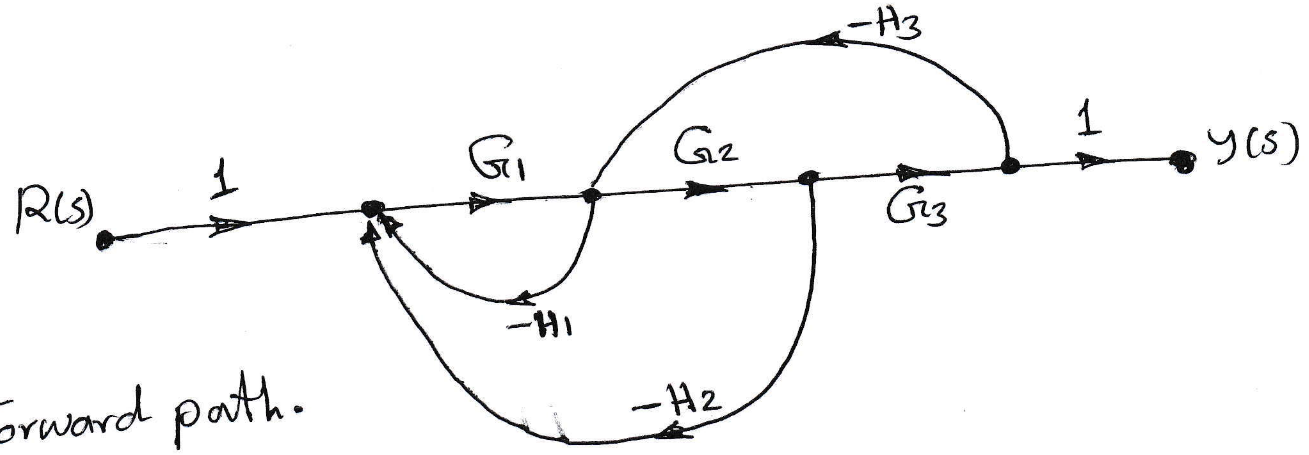
5- the Transfer function is

$$\frac{V_o(S)}{V_i(S)} = \frac{M_1 \Delta_1}{\Delta}$$
$$= \frac{\frac{LCS^2 R_2}{R_1}}{1 + \frac{LS}{R_1} + S^2LC + R_2CS + \frac{R_2}{R_1}LCS^2}$$

$$\frac{V_o(S)}{V_i(S)} = \frac{LCS^2 R_2}{S^2LC(R_1 + R_2) + S(R_1R_2C + L) + R_1}$$

EX Find the transfer function by Mason's gain formula for the block diagram.





1 - Forward path.

$$M_1 = G_1 G_2 G_3$$

2- Loop gain.

$$L_1 = -G_1 H_1$$

$$L_2 = -G_1 G_2 H_2$$

$$L_3 = -G_2 G_3 H_3$$

3- Two or more non touching loops are not present

$$P = 0$$

4- the determinant of the graph.

$$\Delta = 1 + G_1 H + G_1 G_2 H_2 + G_2 G_3 H_3$$

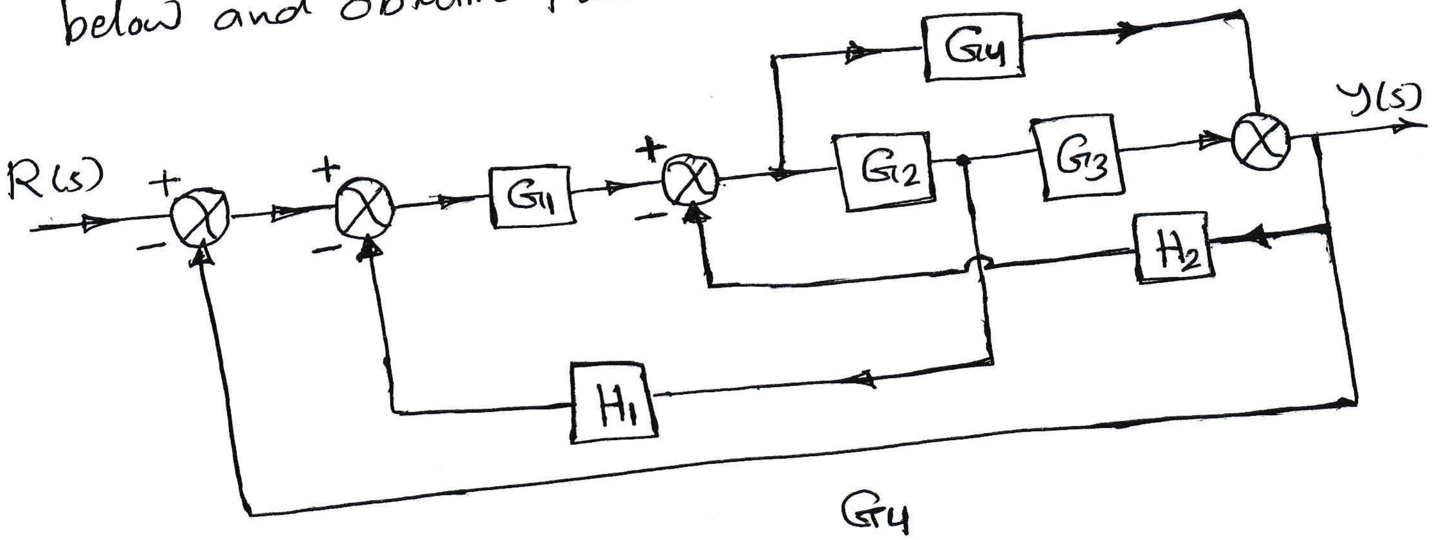
$$\Delta_1 = 1$$

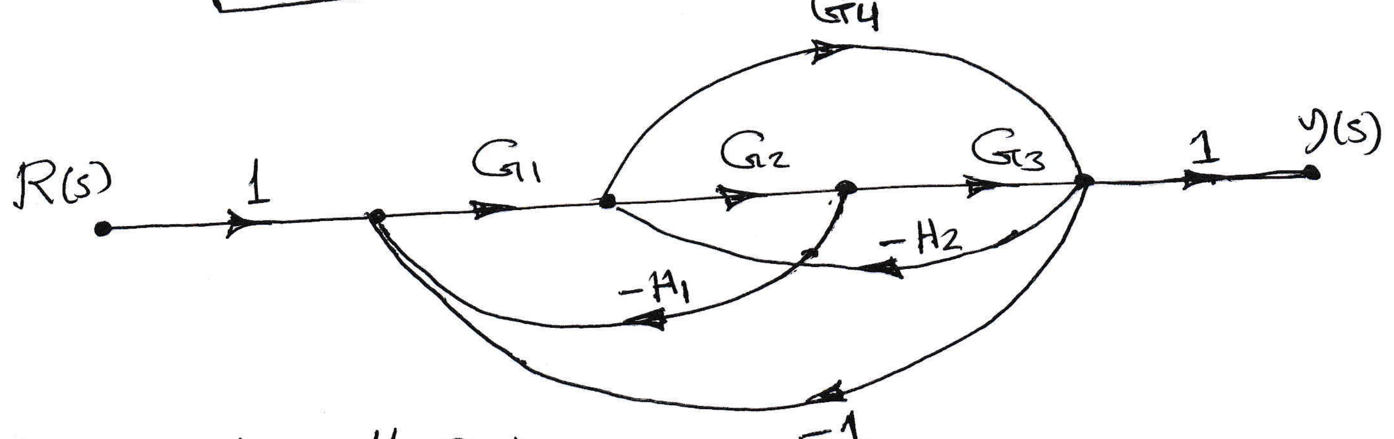
5- Transfer function

$$\frac{Y(s)}{R(s)} = \frac{M_1 \Delta_1}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_1 H_1 + G_1 H_2 + G_2 G_3 H_3}$$

EX
Draw the signal flow graph for the block diagram below and obtain the transfer function $\frac{Y(s)}{R(s)}$.





1 - Forward path gain.

$M_1 = G_1 G_2 G_3$

$M_2 = G_1 G_4$

2- Loop gains.

$$L1 = -G_1 G_2 H_1$$

$$L2 = -G_2 G_3 H_2$$

$$L3 = -G_1 G_2 G_3$$

$$L4 = -G_4 H_2$$

$$L5 = -G_1 G_4$$

3- two or more non touching loops.

$$P = 0$$

4- the determinant of the graph.

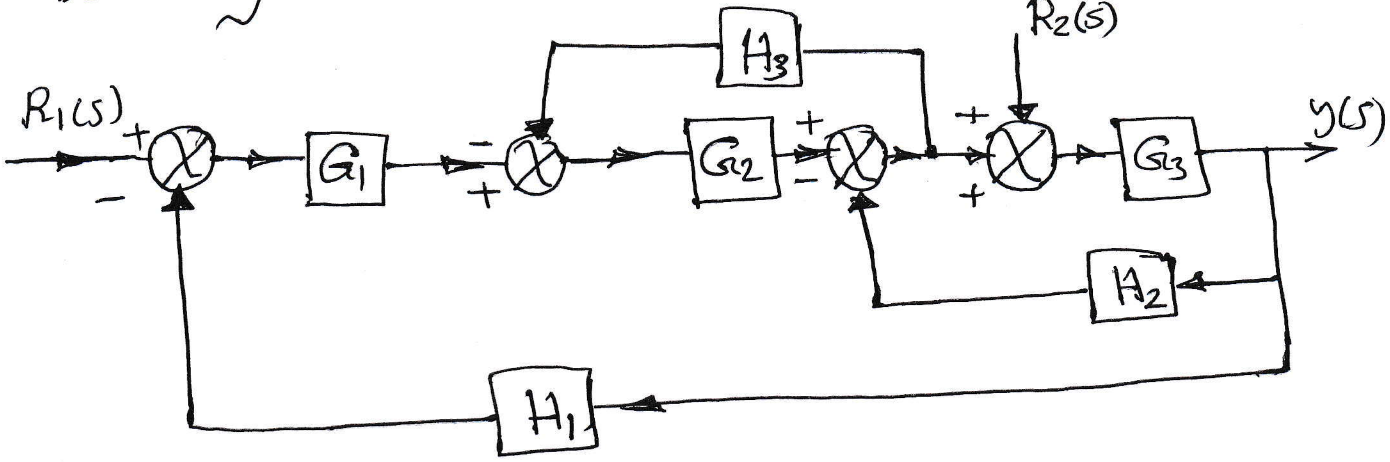
$$\Delta = 1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_4 H_2 + G_1 G_4$$

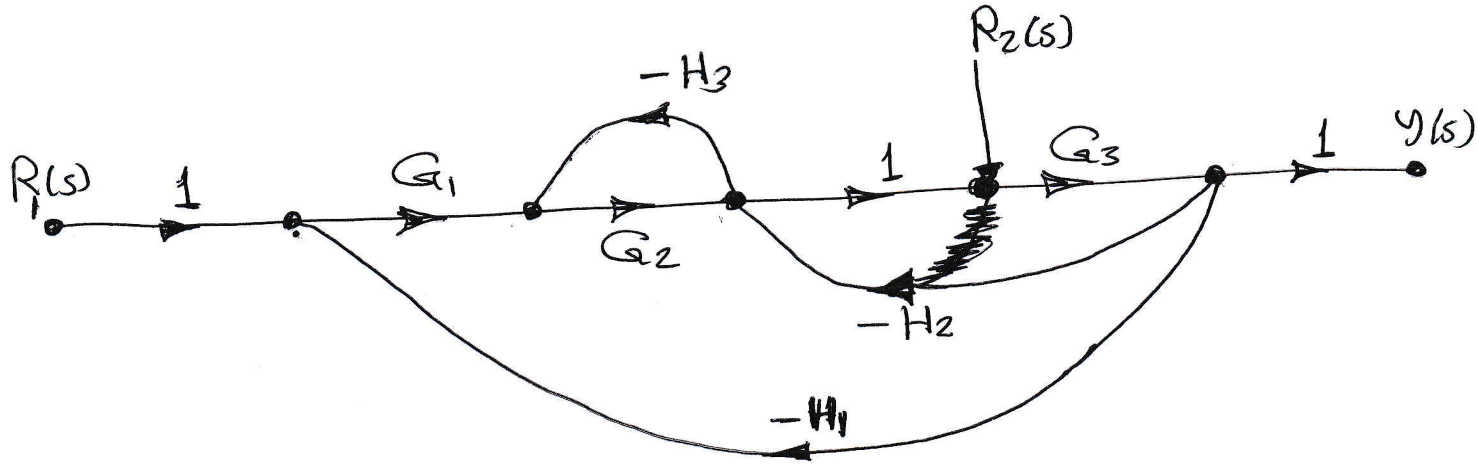
$$\Delta 1 = 1$$
$$\Delta 2 = 1$$

5- The transfer function is,

$$\frac{Y(s)}{R(s)} = \frac{G_1 (G_2 G_3 + G_4)}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_4 H_2 + G_1 G_4}$$

EX Find the transfer function $\frac{Y(s)}{R_1(s)}$ and $\frac{Y(s)}{R_2(s)}$ in figure below by Mason's gains formula.





(a) for Transfer function $\frac{Y(s)}{R_1(s)}$, $R_2(s)=0$

1- forward path gain

$$M_1 = G_1 G_2 G_3$$

2- loop gain

$$L1 = -G_2 H_3$$

$$L2 = -G_3 H_2$$

$$L3 = -G_1 G_2 G_3 H_1$$

3- two or more non touching loops are not present

$P = 0$

4- the determinant of the graph.

$$\Delta = 1 + G_2 H_3 + G_3 H_2 + G_1 G_2 G_3 H_1$$

$$\Delta_1 = 1$$

5- the transfer function is,

$$\frac{Y(S)}{R(S)} = \frac{G_1 G_2 G_3}{1 + G_2 H_3 + G_3 H_2 + G_1 G_2 G_3 H_1}$$

(b) Transfer function $\frac{Y(S)}{R_2(S)}$, $R_1(S) = 0$

1- forward path gains

$$M_1 = G_3$$

2- loop gains

$$L_1 = -G_3 H_2$$

$$L_2 = -G_3 H_1 G_1 G_2$$

$$L_3 = -G_2 H_3$$

3- Two or more nontouching loops are not present, $P = 0$

4- Determinant of the graph

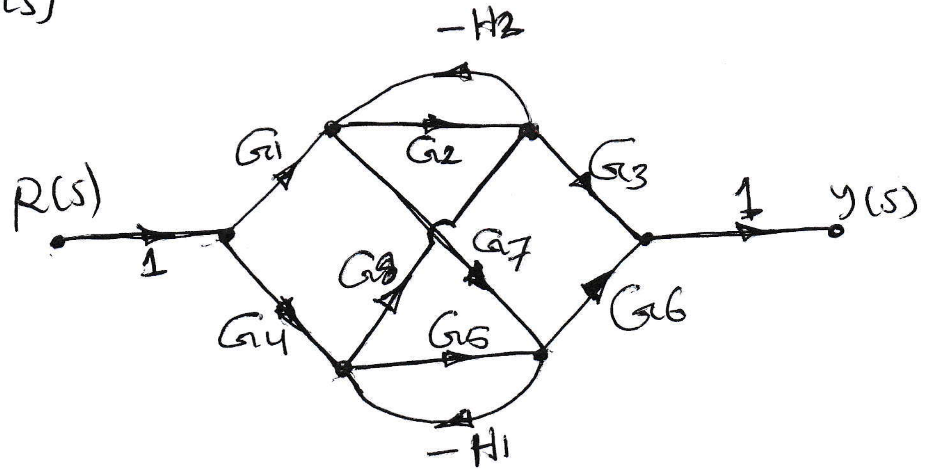
$$\Delta = 1 + G_3 H_2 + G_3 H_1 G_1 G_2 + G_2 H_3$$

$$\Delta_1 = 1 + G_2 H_3$$

5- Transfer function,

$$\frac{Y(s)}{R_2(s)} = \frac{G_3(1 + G_2 H_3)}{1 + G_3 H_2 + G_3 H_1 G_1 G_2 + G_2 H_3}$$

EX obtain $\frac{Y(s)}{R(s)}$ for the signal flow graph.



1- Forword path gains

$$M_1 = G_1 G_2 G_3$$

$$M_2 = G_1 G_7 G_6$$

$$M_3 = -G_1 G_7 H_1 G_8 G_3$$

$$M_4 = -G_4 G_8 H_2 G_7 G_6$$

$$M_5 = G_4 G_5 G_6$$

$$M_6 = G_4 G_8 G_3$$

2- loop gains.

$$L_1 = -G_2 H_2$$

$$L_2 = -G_5 H_1$$

$$L_3 = G_7 H_1 G_8 H_2$$

3- Two non touching loops.

L_1 and L_2 $\Rightarrow$ $P_{12} = G_2 G_5 H_1 H_2$

4- the determinant of the graph is

$$\Delta = 1 + G_2 H_2 + G_5 H_1 - G_7 G_8 H_1 H_2 + G_2 G_5 H_1 H_2$$

$$\Delta_1 = (1 + H_1 G_5)$$

$$\Delta_2 = \Delta_3 = \Delta_4 = \Delta_6 = 1$$

$$\Delta_5 = 1 + H_2 G_2$$

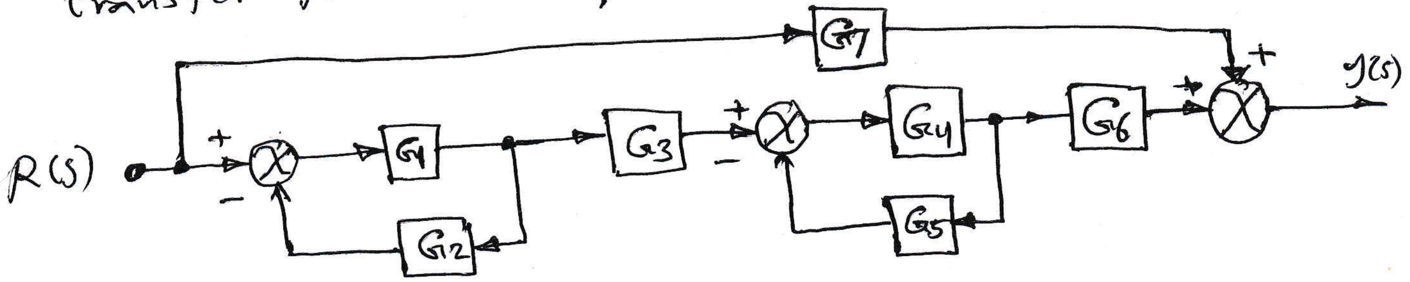
5- The transfer function $\frac{Y(s)}{R(s)}$ is,

$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 (1 + H_1 G_5) + G_1 G_7 G_6 - G_1 G_7 H_1 G_8 G_3 + G_4 G_5 G_6 (1 + H_2 G_2) + G_4 G_8 G_3 - G_4 G_8 H_2 G_7 G_6}{1 + G_2 H_2 + G_5 H_1 - G_7 G_8 H_1 H_2 + G_2 G_5 H_1 H_2}$$

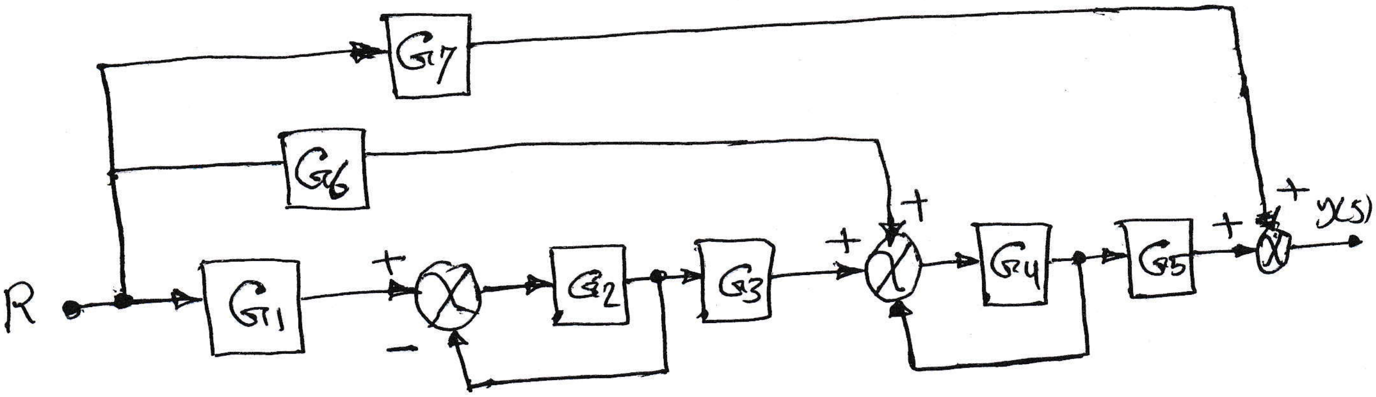
Block diagram   Homework 2

Q1 Reduction the block diagrams below and find the transfer function $Y(s)/R(s)$.

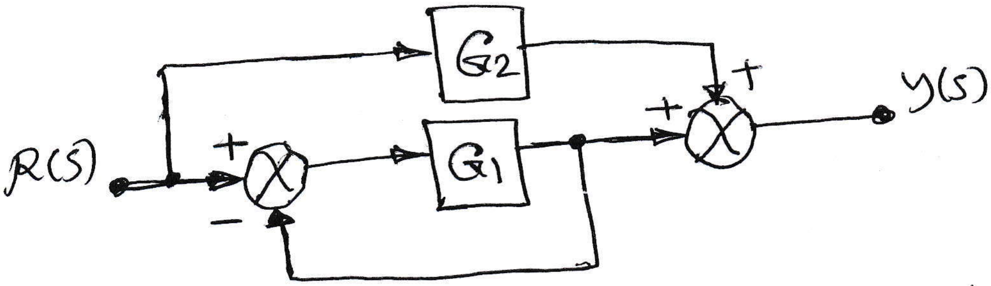
(a)



(b)

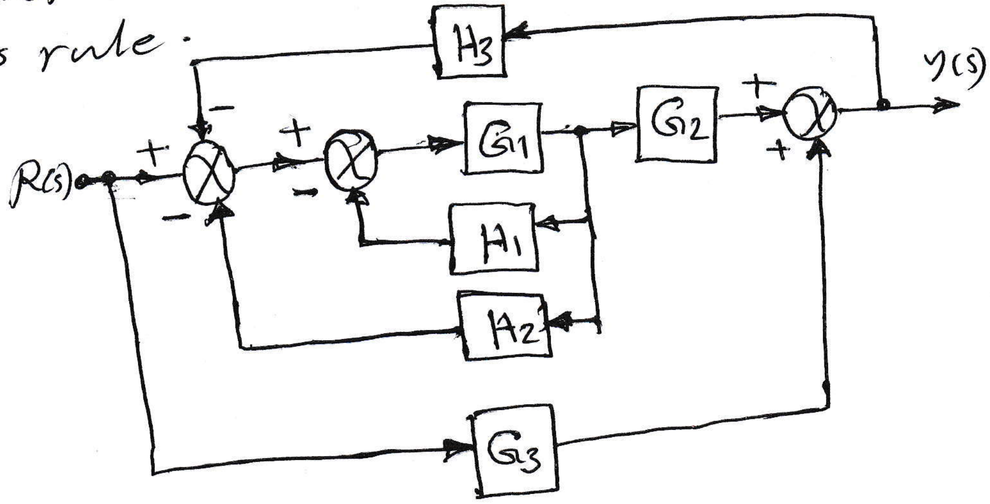


(c)

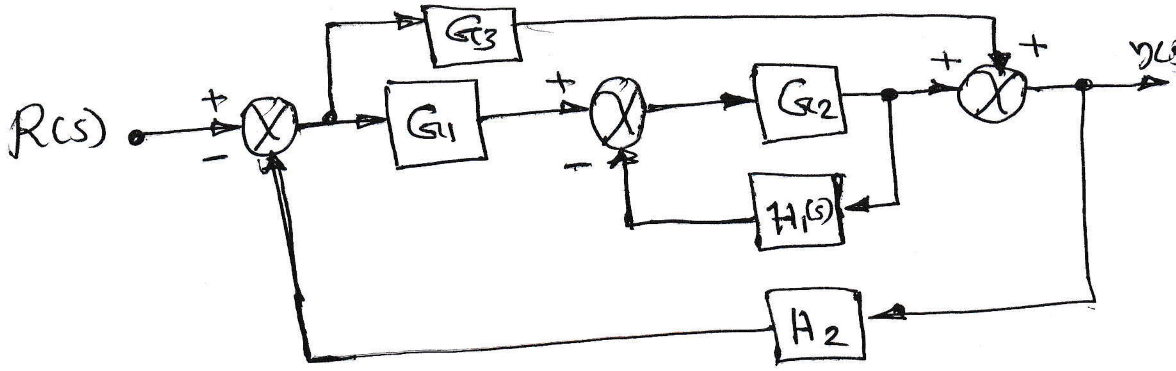


Q2 Find the transfer function for the block diagrams using Mason's rule.

(a)



b)



Q3 Use block-diagram algebra and Mason's rule to determine the transfer function between R(s) and Y(s) in figure below.